# On the Identification of DSGE Models

Ivana Komunjer* Serena Ng†

March 2009

**Abstract**

This paper provides necessary and sufficient conditions for identifying the parameters of a DSGE model. These conditions do not depend on the choice of the estimator and should be checked before estimation. The conditions provide an upper bound for how many free parameters can be estimated, and by implication, establishes the minimum number of parameters that must be held fixed. A crucial part of the analysis is to determine whether the solution of the model can be identified from the state space representation of the solution. This can verified from the properties of the Markov parameters of the state space model. Fixing the correct number of parameters is necessary but not sufficient for identification. We also propose two indicators to check for "conditional identification." We show that even in a simple stochastic growth model, identification can fail even when the parameters are not on the boundary of the parameter space.

[Preliminary and Incomplete. Please do not circulate.]

*Department of Economics, University of California San Diego, 9500 Gilman Drive MC 0508, La Jolla CA 92093-0508. E-mail: komunjer@ucsd.edu.

†Department of Economics, Columbia University, 420 West 118 Street MC 3308, New York, NY 10027. Email: serena.ng@columbia.edu

Ng acknowledges Financial support from the NSF under grant SES-0549978. This work was done while Komunjer was visiting Columbia University whose hospitality is gratefully acknowledged.

# 1 Introduction

Dynamic stochastic general equilibrium (DSGE) models have now reached the level of sophistication to permit analysis of important policy and theoretical macroeconomic issues. Whereas the parameters in these models used to be calibrated, numerical advances in the last two decades have made it possible to estimate models with as many as a hundred of parameters. While guided mostly by intuition, researchers are aware that not all model parameters can be identified, a procedure has yet to exist that tells us in a systematic manner how many parameters are identifiable. This paper establishes the necessary (order) and sufficient (rank) conditions for identification in DSGE models. We show how to determine the maximum number of parameters that can be identified, which by implication, also determines the minimum number of parameters that needs to be fixed.

In a typical DSGE analysis, one starts by writing down an optimizing model expressed in terms in the so-called deep parameters, say $\theta$. One then log-linearizes the model and this log-linearized model has structural parameters, $\gamma$. The structural model is 'solved', and the model solution is expressed in terms of solution parameters, $\lambda$. The model is then put to a state space form whose parameters can be denoted $\pi$. The parameter vector of interest is typically $\theta$ or $\gamma$. Identification can fail when (i) $\pi$ is uninformative about $\lambda$, and/or (ii) $\lambda$ is uninformative about $\gamma$, and/or (iii) $\gamma$ is uninformative about $\theta$. If identification fails at any one step, no estimator will yield consistent estimates of the true parameters. This means that whether we perform Euler equation estimation as in Christiano and Eichenbaum (1992) and Fukac and Pagan (2007), or covariance structure estimation as in Christiano and den Haan (1996), or the two-step minimum distance estimator of Sbordonne (2006), or by maximum likelihood as in Ireland (1997), the parameters of interest will not be identified. Bayesian estimation as in An and Schorfheide (2007) is increasingly used in recent years. Introducing a prior changes the sample objective function but does not resolve the identification problem at the population level per se. As Poirier (1998) pointed out, if the parameter space is not variation free, the prior on the non-identified parameters may be updated because of the parameter constraints imposed by the model. When the prior is not properly marginalized, the posterior and the prior can differ, but this does not mean that the parameters are identified.

Our results shed light on why almost every empirical DSGE exercise estimates only a subset of the parameters and fixes many others. For example, Del Negro, Schorfheide, Smets, and Wouters (2007) fix 7 of the 47 model parameters, while Smets and Wouters (2007) fix 7 of the 39 parameters. Christiano, Motto, and Rostagno (2007) split the model parameters into a set of 26 parameters that controls the steady state, and which they fixed at values taken from the literature. They then estimate the remaining 55 parameters that control the dynamics. Even with the simple stochastic growth model, Ruge-Murcia (2007) only estimates three of the six parameters and fixes three parameters. While one might think that identification failure arises only with complex models, or when parameters are on the boundary of their support, we show using a simple example that this is not the case. Furthermore, fixing the 'correct' number of parameters still does not ensure identification of the remaining parameters. We show how to check for "conditional identification."

Identification failure is often associated with a lack of curvature in the second derivative (or the Hessian) of the likelihood function. In extremum estimation where $Q_0(\theta_0)$ is the population objective function evaluated at the true parameter vector $\theta_0$, global identification requires that $Q_0(\theta_0) > Q_0(\theta)$ for any $\theta$ in the parameter space $\Theta$. Local identification requires that $Q_0(\theta_0) \neq Q_0(\theta)$ for $\theta$ in the neighborhood of $\theta_0$. Or, in other words, that $Q_0(\theta)$ is not flat in any dimension of $\theta_0$. Canova and Sala (2006) suggest to check whether the Hessian of $Q_n$ is singular, where $Q_n$ is the sample analog of $Q_0$. Consolo, Favero, and Paccagnini (2007) suggest to check for statistical identification by comparing the properties of the VAR implied by the DSGE model with those of a factor augmented VAR, also implied by the DSGE model. Both approaches rely on estimates of $\theta$, denoted $\hat{\theta}$. But when identification failure fails, $\hat{\theta}$ is not consistent for $\theta_0$. Even if procedures can detect the symptoms, they cannot be precise when identification fails.

Identifiability of the parameters is a population problem and should be checked *before* estimation. Iskrev (2007, 2008) observe that the negative of the expected value of the Hessian is the information matrix. It follows that the asymptotic covariance of the deep parameters can be expressed in terms of (i) the asymptotic covariance of the reduced form model, and (ii) the gradient that maps the model to the reduced form parameters. The former determines how well the reduced form parameters are identified, while the latter determines how well the deep (or structural) parameters $\theta$ (or $\gamma$) are identified. Assuming that the covariance matrix of the reduced model is well determined from the data, these two papers and focus on checking for rank deficiency of gradient matrix, which does not depend on the data. Our main point of departure is that the identifiability of the reduced form parameters cannot be taken as granted. The condition must be checked. As we will show, this involves checking the properties of impulse responses via the Markov parameters of the state space model.

Confounding the sample with the population problem unnecessarily detours the issue to 'weak identification'. Such problems arise when, in the present context, the data are uninformative about $\lambda$. But this is a problem of second order importance when $\lambda$ is not even identifiable in the population. The order condition that we will establish is based on simple counting of the number of parameters that can be estimated from the state space model, vis-́-vis the dimension of the reduced form parameter vector, $\lambda$. While this condition is necessary, it is not sufficient. It is only when an additional rank condition holds that the mapping from the state space model to the reduced form parameters is unique. Once identification of $\lambda$ is granted, we can proceed to check rank and order conditions for the mapping from the structural to the reduced form model, and finally the conditions for the mapping from the structural model to the deep parameters. It is only when the rank and order conditions are satisfied at all three steps that the deep parameters can be identified in the population. Ultimately, the maximum number of parameters we can identify is the minimum of all the identifiable parameters at each mapping. This in turn determines the number of parameters that need to be fixed.

Our identification analysis will be studied in the context of a simple stochastic model, which will be presented in Section 2. The basic identification approach will be given in Section 3. Section 4 applies the analysis to three versions of the DSGE model. Section 5 focuses on identification of the solution parameters from the state space model using Markov parameters. Section 6 concludes.

## References


An, S., and F. Schorfheide (2007): "Bayesian Analysis of DSGE Models," *Econometric Reviews*, 26: 2-4, 113–172.

Blanchan, O., and C. Kahn (1980): "The Solution of Linear DIfference Models under Rational Expectatons," *Econometrica*, 48, 1305–1313.

Canova, F., and L. Sala (2006): "Back to Square One: Identification Issues in DSGE Models," *Journal of Monetary Economics*, forthcoming.

Christiano, L., and W. den Haan (1996): "Small Sample properties of GMM for Business-Cycle Analysis," *Journal of Business and Economic Statistics*, 14, 309–327.

Christiano, L., and M. Eichenbaum (1992): "Liquidity Effects and the Monetary Transmissions Mechanisms," *American Economic Review*, 82:2, 346–353.

Christiano, L., R. Motto, and M. Rostagno (2007): "Financial Factors in Business Cycles," manuscript, Northwestern University.

Consolo, A., C. Favero, and A. Paccagnini (2007): "On the Statistical Identification of DSGE Models," IGIER Working Paper n.324.

Del Negro, M., F. Schorfheide, F. Smets, and R. Wouters (2007): "On the Fit of New Keynesian Models," *Journla of Business and Economic Statistics*, 25:2, 143–162.

Fernandez-Villaverde, F., J. Rubio-Ramirez, T. Sargent, and M. Watson (2007): "A,B,Cs and (D)'s for Understanding VARs," *American Economic Review*, 97:3, 1021–1026.

Fukac, M., and A. Pagan (2007): "Limited Information Estimation and Evaluation of DSGE models," *Journal of Applied Econometics*, forthcoming.

Glover, K., and J. Willems (1974): "Parameterizations of Linear Dynamical Systems: Canonical Forms and Identifiability," *IEEE Transactions on Automatic Control*, 19:6, 640–646.

——— (1976): "Identifiability of Linear and Nonlinear Dynamic Systems," *IEEE Transactions on Automatic Control*, 21:6, 833–837.

Hannan, E., and M. Diestler (1988): *The Statistical Theory of Linear Systems*. Wiley, New York.

Ireland, P. (1997): "A Small Structural Quarterly Model for Monetary Policy Evaluaton," *Carnegie-Rochester Conference Series on Public Policy*, 47, 83–108.

Iskrev, N. (2007): "Evaluating the Information Matrix in Linearized DSGE models," *Economics Letters*, 99:3, 607–610.

——— (2008): "How Much Do We Learn from Estimation of DSGE models? A Case Study of Identification Issues in a New Keynesian Business Cycle Model," University of Michigan.

Poirier, D. (1998): "Revisign Beliefs in Nonidentified Models," *Econometric Theory*, 14, 183–209.

Rothenberg, T. (1971): "Identification in Parametric Models," *Econometrica*, 39:3, 577–591.

Ruge-Murcia, F. J. (2007): "Methods to Estimate Dynamic Stochastic General Equilibrium Models," *Journal of Economic Dynamics and Control*, 31, 2599–2636.

Sbordonne, A. (2006): "U.S. Wage and Price Dynamics: A Limited Information Approach," *International Journal of Central Banking*, 2:3, 155–191.

Smets, F., and R. Wouters (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *The American Economic Review*, 97:3, 586–606.

Uhlig, H. (1999): "A Toolkit for Analyzing Nonlinear Dynamic Stochastic Models Easily," in *Computational Methods for the Study of Dynamic Economies*, ed. by R. Marimon, and A. Scott, pp. 30–61. Oxford University Press.